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TABLE OF CONTENTS

	Page
BINOMIAL THEOREM	1
CALCULUS	1
EXPRESSIONS	2
GEOMETRY—SECTION I	2
GEOMETRY—SECTION II	3
INDICES (EXPONENTS)	4
LOGARITHMIC FUNCTIONS	5
MATRICES	5
NUMBER SYSTEMS	6
PERMUTATIONS AND COMBINATIONS	6
POLYNOMIALS OF HIGHER DEGREE	7
PROBABILITY	7
RELATIONS AND FUNCTIONS	8
SEQUENCES AND SERIES	9
SYSTEMS OF LINEAR EQUATIONS	10
TRIGONOMETRIC FUNCTIONS	10
VECTORS	11

All variables in this booklet are real numbers unless otherwise specified.

BINOMIAL THEOREM

TERM	DEFINITION
1. General Term of $(x + y)^n$ is	$\binom{n}{r}x^{n-r}y^r$

CALCULUS

TERM	DEFINITION
1. AVERAGE VELOCITY:	The ratio of the total directed distance travelled in that time to the total time interval.
2. INSTANTANEOUS VELOCITY:	The instantaneous velocity (v) of a body that has travelled a distance (s) in a time (t) is given by $v = \lim_{\triangle t \to 0} \frac{\triangle s}{\triangle t} = D_t s$
3. AVERAGE ACCELERATION:	The ratio of the rate of change in velocity over a given time interval to the time interval itself; average acceleration (in interval $\triangle t$) $= \frac{\triangle v}{\triangle t}$, where $\triangle v$ is change in velocity, over the interval $\triangle t$.
4. INSTANTANEOUS ACCELERATION:	$a = \lim_{\triangle t \to 0} \frac{\triangle v}{\triangle t} = D_t v = D^2_t s$ where a = acceleration, v = velocity, s = distance, t = time
5. DERIVATIVES:	(a) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x) = D_x y = \frac{dy}{dx} = y^1$, where $y = f(x)$
	(b) if $y = cx^n$, then $D_x y = cnx^{n-1}$, where c, $x \epsilon R_e$
	(c) Chain Rule: $D_x F(x) = D_u Y \cdot D_x U$ *OR* $D_x Y = D_u Y \cdot D_x U$
	(d) Product: $D_x y = vD_x U + uD_x v$
	(e) Division: $D_x y = \frac{vD_x u - uD_x v}{v^2}$
6. PRIMITIVE (anti-derivative *OR* integral) of $D_x y = x^n$ is	$y = \frac{x^{n+1}}{n+1} + c$, where c = constant
7. AREA:	$aA_b = F(b) - F(a)$, where A = area under curve and a and b = limits of area

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EXPRESSIONS

TERM	DEFINITION
1. REAL ZERO OF A POLYNOMIAL:	That real number replacement of the variable which results in the value of the polynomial becoming zero.
2. FACTORING OF POLYNOMIALS:	
(a) $P^2 + 2PQ + Q^2$ equals	$(P + Q)^2$
(b) $P^2 - Q^2$ equals	$(P + Q)(P - Q)$
(c) $P^2 - 2PQ + Q^2$ equals	$(P - Q)^2$
(d) $P^3 + Q^3$ equals	$(P + Q)(P^2 - PQ + Q^2)$
(e) $P^3 - Q^3$ equals	$(P - Q)(P^2 + PQ + Q^2)$

GEOMETRY – SECTION 1

TERM	DEFINITION
1. DISTANCE BETWEEN POINTS ON A PLANE:	For two points (a, b) and (c, d) the distance between the points is given by $\sqrt{(a - c)^2 + (b - d)^2}$
2. SLOPE OF A LINE:	For a line passing through the points (x_1, y_1) and (x_2, y_2) the slope of the line (m) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$
3. POINT-SLOPE FORM OF A LINE:	$y - y_1 = m(x - x_1)$
4. MID-POINT OF A LINE SEGMENT:	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
5. CIRCLE:	(a) $(x - h)^2 + (y - k)^2 = r^2$ where r is radius, (h, k) is centre. (b) $x^2 + y^2 + Dx + Ey + F = r$, $r = \frac{1}{2}\sqrt{D^2 + E^2 - 4F}$, centre $= \left(\frac{-D}{2}, \frac{-E}{2}\right)$
6. PARABOLA CENTRE AT ORIGIN:	(a) $x^2 = 4py$ or $y^2 = 4px$ (b) $y = ax^2 + bx + c$
7. ELLIPSE CENTRE AT ORIGIN:	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $a^2 = b^2 + c^2$ where a = half major axis, b = half minor axis
8. HYPERBOLA CENTRE AT ORIGIN:	(a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $c^2 = a^2 + b^2$ (b) $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

GEOMETRY – SECTION 2

TERM	DEFINITION
1. MEDIAN OF A TRIANGLE:	A line segment whose endpoints are a vertex and the midpoint of the opposite side.
2. SKEW LINES:	Noncoplanar lines.
3. DIHEDRAL ANGLE:	A three-dimensional angle formed from the union of two half-planes.
4. POWER OF A POINT WITH RESPECT TO A CIRCLE:	Given C(P, r), point A, and any line through A intersecting the circle in points B and C; the power of A with respect to C(P, r) is the product of d(A, B) and d(A, C) where d(A, B) and d(A, C) represent the distance from A to B and from A to C, respectively.

5. FORMULAS:

(a) Area

circle	πr^2
square	s^2
rectangle	lw
triangle	$\frac{1}{2}bh$
parallelogram	lh
trapezium	$\frac{h}{2}(a + b)$

(b) Perimeter or Circumference

circle	$2\pi r$ or πd
square	$4s$
rectangle	$2(l + w)$

(c) Volume

cube	lwh
sphere	$\frac{4}{3}\pi r^3$
cylinder	$\pi r^2 h$
cone	$\frac{1}{3}\pi r^2 h$

6. THEOREM 1: Two triangles are congruent if there is a congruence between

(a) 2 sides and the included angle in each triangle (S.A.S.)

(b) 2 angles and one side in each triangle (A.S.A.)

(c) the three sides in each triangle (S.S.S.)

TERM	DEFINITION
7. THEOREM 2:	Two right triangles are congruent if the hypotenuse and a leg of one triangle are congruent to the hypotenuse and leg of the other triangle.
8. THEOREM 3:	Two triangles are similar if their corresponding angles are congruent.
9. THEOREM 4:	The sum of the measures of the angles of a convex polygon of n sides is $(n - 2)180$.
10. THEOREM 5:	The length (ℓ) of an arc of radius r and measure m is $\ell = \frac{m\pi r}{180}$.
11. THEOREM 6:	Angles inscribed in congruent arc are congruent.
12. THEOREM 7:	The measure of an intercepted arc is the measure of the central angle.
13. THEOREM 8:	The measure of an inscribed angle is one-half the measure of the intercepted arc.
14. THEOREM 9:	When two parallel lines are intercepted by a transversal (a) alternate interior angles are congruent (b) alternate exterior angles are congruent (c) corresponding angles are congruent

INDICES (EXPONENTS)

TERM	DEFINITION
1. INDICES:	$x^n = x \cdot x \cdot x \ldots x$ where "x" is used "n" times, $n \in N$
2. LAWS OF INDICES:	
(a) $x^0 = 1, x \neq 0$	
(b) $x^{-n} = \frac{1}{x^n}, x \neq 0, n \in I^+$	
(c) $x^r \cdot x^s = x^{r+s}, r, s \in I$	
(d) $(x^r)^s = x^{rs}, r, s \in I$	
(e) $(x \cdot y)^k = x^k \cdot y^k, k \in I$	
(f) $\left(\frac{x}{y}\right)^k = \frac{x^k}{y^k}, y \neq 0, k \in I$	
(g) $\frac{x^r}{x^s} = x^{r-s}, r, s \in I$	

TERM	DEFINITION
(h) $x^{\frac{1}{n}} = \sqrt[n]{x}$, $n \epsilon N$	
(i) $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$, $m, n \epsilon N$	
3. ORDER OR OPERATIONS:	
(a) $-x^k$ means $-(x^k)$	
(b) xy^k means $x(y^k)$	

LOGARITHMIC FUNCTIONS

TERM	DEFINITION
1. LOG OF A PRODUCT:	$\log(m \times n) = \log(m) + \log(n)$ for all positive real numbers m and n.
2. LOG OF A QUOTIENT:	$\log\left(\frac{m}{n}\right) = \log(m) - \log(n)$ for all positive real numbers m and n.
3. LOG OF POWERS:	$\log(m^c) = c \cdot \log(m)$ for all positive real numbers m and n and for each natural number c.

MATRICES

TERM	DEFINITION
1. ROW-REDUCED ECHELON FORM OF A MATRIX MUST MEET THESE CONDITIONS:	(a) the first nonzero entry of each row is 1 (b) each row contains more initial zero terms than the preceding row
2. ADDITION:	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} f & g \\ h & i \end{bmatrix}$, then $A + B = \begin{bmatrix} a + f & b + g \\ c + h & d + i \end{bmatrix}$.
3. MULTIPLICATION:	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $kA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$.
4. TRANSFORMATION ONTO A PLANE:	The mapping of the transformation T provides a one-to-one correspondence between the points of the xy-plane and the points of the uv-plane. T is a one-to-one mapping of the xy-plane onto the uv-plane.
5. TRANSFORMATIONS INTO A PLANE:	This transformation puts all points of the xy-plane into a specific pattern, like a line, on the uv-plane. This is not a transformation of a plane onto a plane, but rather a transformation of the plane into a plane.

TERM	DEFINITION
6. INVERSE TRANSFORMATION:	The transformation $T\begin{cases} u = ax + by \\ v = cx + dy \end{cases}$ has an inverse T^{-1} if and only if $ad - bc \neq 0$.

NUMBER SYSTEMS

SYMBOL	DEFINITION
1. N	natural numbers $\{1, 2, 3, 4, \ldots\}$
2. W or N_0	whole numbers $\{0, 1, 2, 3, 4, \ldots\}$
3. I	set of integers
4. Q	rational numbers
5. R or Re	real numbers
6. C	complex numbers
7. $a + bi$ or (a, b)	standard form of a complex number
8. i	represents $\sqrt{-1}$

TERM	DEFINITION
1. PRIME NUMBER:	A natural number which has exactly two natural number divisors. e.g. $\{2, 3, 5, 7, \ldots\}$
2. COMPOSITE NUMBER:	A natural number which has more than two natural number divisors. e.g. $\{4, 6, 8, 9, \ldots\}$
3. MULTIPLICATIVE INVERSE IN COMPLEX NUMBERS:	The multiplicative inverse of (a, b) or $a + bi$ is $\frac{a}{a^2 + b^2} + \frac{-b}{a^2 + b^2} i$
4. ABSOLUTE VALUE (MODULUS) OF A COMPLEX NUMBER:	The real number $\sqrt{a^2 + b^2}$ is the absolute value of the complex number (a, b) or $(a + bi)$; it is indicated by the symbol $\lvert a + bi \rvert$.

PERMUTATIONS AND COMBINATIONS

TERM	DEFINITION
1. n! equals	$n(n - 1)(n - 2)(n - 3) \ldots 3 \times 2 \times 1$
2. CIRCULAR (distinguishable positions):	$n!$
3. CIRCULAR (indistinguishable positions):	$(n - 1)!$

TERM	DEFINITION
4. ${}_nP_k$ or $(n)_k$ equals	$\frac{n!}{(n - k)!}$
5. PERMUTATIONS (unlike elements):	$\frac{n!}{p!q!r! \ldots}$
6. ${}_nC_k$ or $\binom{n}{k}$ equals	$\frac{n!}{(n - k)!k!}$

POLYNOMIALS OF HIGHER DEGREE

TERM	DEFINITION
1. POLYNOMIAL FUNCTION:	A set of ordered pairs $(x, f(x))$ where $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$. The natural number exponent n, $a_0 \neq 0$, states the **degree** of the function.
2. REMAINDER THEOREM:	If a polynomial $P(x)$ is divided by $(x - a)$, the remainder is given by $P(a)$.
3. ZERO OF A POLYNOMIAL:	The number "r" is called a **zero** of a polynomial $P(x)$ if and only if $P(r) = 0$.
4. FACTOR THEOREM:	Given a polynomial $P(x)$ such that $P(r) = 0$, then $(x - r)$ is a factor of $P(x)$. *OR* The binomial $(x - r)$ is a factor of the polynomial $P(x)$ if and only if $P(r) = 0$.
5. INTEGRAL ZERO THEOREM:	If an integer "r" is a zero of an integral polynomial $P(x)$, then "r" is a factor of the constant term of $P(x)$.
6. RATIONAL ZERO THEOREM:	If $\frac{a}{b}$ ($b \neq 0$, a and b are relatively prime integers) is a zero of the integral polynomial $P(x) = a_0x^n + ax^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, then "a" is a divisor of a_n and b is a divisor of a_0.
7. CONJUGATE ZERO THEOREM:	If $a + bi$ is a zero of a real polynomial $P(x)$, then $a - bi$ is also a zero of $P(x)$. Thus complex zeros occur in conjugate pairs.

PROBABILITY

TERM	DEFINITION
1. P(E) equals	$\frac{n(E)}{n(S)}$, where E = favorable outcomes S = total possible outcomes

TERM	DEFINITION
2. $P(E \cup F)$ equals	$P(E) + P(F) - P(E \cap F)$ Addition theorem — P(E or F)
3. $P(E \cap F)$ equals	$P(F) \times P(E\|F)$ $P(E) \times P(F\|E)$, where E and F are particular events.

RELATIONS AND FUNCTIONS

TERM	DEFINITION
1. RELATION:	Any set of ordered pairs.
2. A′	Inverse of the relation A.
3. FUNCTION:	A relation F where no two ordered pairs in F have the same first component.
4. FUNCTION NOTATIONS WHICH ARE EQUIVALENT:	(a) $f(x) = 2x + 1$ (b) $f = \{(x, y) \mid y = 2x + 1\}$ (c) $f = \{(x, 2x + 1)\}$ (d) $f: x \to 2x + 1$
5. f^{-1}	The inverse function of the function f.
6. DIRECT PROPORTION FUNCTION:	$\{(x, y) \mid y = cx, x \epsilon D\}$ where D is the domain of the function. "c" is the constant of proportionality.
7. INVERSE PROPORTION FUNCTION:	$\{(x, y) \mid y = \frac{c}{x}, x \epsilon D\}$ for every real number $c \neq 0$ where D is the domain of the function. "c" is the constant of proportionality.
8. COMPLETED SQUARE FORM of $ax^2 + bx + c$ where a, b, c and x are real numbers and $a \neq 0$	$ax^2 + bx + c = a\left[x - \frac{-b}{2a}\right]^2 + \left(\frac{4ac - b^2}{4a}\right)$
9. QUADRATIC FORMULA FOR SOLVING $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
10. SUM OF THE ROOTS FOR GENERAL QUADRATIC EQUATION $ax^2 + bx + c = 0, a \neq 0$	$\frac{-b}{a}$
11. PRODUCT OF THE ROOTS FOR GENERAL QUADRATIC EQUATION $ax^2 + bx + c = 0, a \neq 0$	$\frac{c}{a}$

SEQUENCES AND SERIES

TERM	DEFINITION
1. GENERAL TERM OF ARITHMETIC PROGRESSION:	$a_n = a_1 + (n - 1)d$, where a is the first term and d equals common difference.
2. SUM OF AN ARITHMETIC SERIES:	$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$, where a is the first term and d equals common difference.
3. GENERAL TERM OF A GEOMETRIC PROGRESSION:	$a_n = a_1 r^{n-1}$, where a is the first term and r equals common ratio.
4. SUM OF A GEOMETRIC SERIES:	$S_n = \frac{a_1(r^n - 1)}{r - 1}$, where a is first term and r equals common ratio.
5. SUMMATION NOTATION:	$\sum$
6. PROPERTIES OF CONVERGENT SEQUENCES:	(a) $\lim_{n\to\infty}(a_n)$ = limit of a_n as n approaches infinity
	(b) $\lim_{n\to\infty}(ka_n) = k \cdot \lim_{n\to\infty} a_n$
	(c) $\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty} a_n + \lim_{n\to\infty} b_n$
	(d) $\lim_{n\to\infty}(a_n b_n) = \lim_{n\to\infty} a_n \cdot \lim_{n\to\infty} b_n$
	(e) $\lim_{n\to\infty}(a_n - b_n) = \lim_{n\to\infty} a_n - \lim_{n\to\infty} b_n$
	(f) $\lim\left(\frac{a_n}{b_n}\right) = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$, $\lim b_n \neq 0$
	(g) limit of a constant "c" $\lim_{n\to\infty} c = c$
	(h) $\lim_{n\to\infty}\left(\frac{1}{n}\right) = 0$
7. SUM OF AN INFINITE GEOMETRIC SERIES:	$S = \frac{a_1}{1 - r}$, where a is the first term and r equals common ratio.

SYSTEMS OF LINEAR EQUATIONS

TERM	DEFINITION
1. INDEPENDENT SYSTEM:	A system of two linear equations which has exactly one ordered pair of numbers in its solution set.
2. INCONSISTENT SYSTEM:	A system of two linear equations which has the empty set for its solution set.
3. DEPENDENT SYSTEM:	A system of two linear equations which has an infinite number of ordered pairs in its solution set.

TRIGONOMETRIC FUNCTIONS

TERM	DEFINITION
1. COSINE FUNCTION (cos):	(a) Wrapping Function Definition $\cos(\theta) = x$ for $W(\theta) = (x, y)$ (b) Right Triangle Definition $\cos(\theta) = \frac{\text{side adjacent to } \angle\theta}{\text{hypotenuse}}$
2. SINE FUNCTION (sin):	(a) Wrapping Function Definition $\sin(\theta) = y$ for $W(\theta) = (x, y)$ (b) Right Triangle Definition $\sin(\theta) = \frac{\text{side opposite to } \angle\theta}{\text{hypotenuse}}$
3. TANGENT FUNCTION (tan):	(a) Wrapping Function Definition $\tan(\theta) = \frac{y}{x}, x \neq 0$ for $W(\theta) = (x, y)$ (b) Right Triangle Definition $\tan(\theta) = \frac{\text{side opposite to } \angle\theta}{\text{side adjacent to } \angle\theta}$
4. TRIGONOMETRIC THEOREMS OF IDENTITIES FOR ALL REAL NUMBERS A AND B:	(a) $\cos^2(A) + \sin^2(A) = 1$ (b) $\tan(A) = \frac{\sin(A)}{\cos(A)}, \cos(A) \neq 0$ (c) $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$ (d) $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ (e) $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ (f) $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ (g) $\cos(-A) = \cos(A)$ (h) $\sin(-A) = -\sin(A)$ (i) $\sin\left(\frac{\pi}{2} - A\right) = \cos(A)$ (j) $\cos\left(\frac{\pi}{2} - A\right) = \sin(A)$

TERM	DEFINITION
5. LAW OF SINES:	For all triangles ABC, if a, b, c are the lengths of the sides opposite the angles A, B, C, respectively, and the measures of angles A, B, C are α°, β° and γ°, respectively, then $\frac{A}{\sin(\alpha^\circ)} = \frac{B}{\sin(\beta^\circ)} = \frac{C}{\sin(\gamma^\circ)}$.
6. LAW OF COSINES:	For all triangles ABC, if a, b and c are the lengths of the sides opposite angles A, B and C, respectively, and the measure of $\angle A$ is α°, then $a^2 = b^2 + c^2 - 2bc\cos(\alpha^\circ)$.

VECTORS

TERM	DEFINITION
1. STANDARD POSITION:	A vector whose initial point is the origin.
2. RECTANGULAR FORM:	Specifies x-component and y-component; rectangular form for the vector $\overrightarrow{AB}$, where $\vec{A} = [a, b]$ and $\vec{B} = [c, d]$ is given by $[c - a, d - b]$.
3. MAGNITUDE:	The magnitude of vector $\overrightarrow{AB}$ is indicated by $\lvert\overrightarrow{AB}\rvert$ (a) if $\overrightarrow{AB} = [x, y]$, then $\lvert\overrightarrow{AB}\rvert = \sqrt{x^2 + y^2}$ (b) if $\overrightarrow{AB} = [x, y, z]$, then $\lvert\overrightarrow{AB}\rvert = \sqrt{x^2 + y^2 + z^2}$
4. DOT PRODUCT:	(a) $\vec{A} \cdot \vec{B} = \lvert\vec{A}\rvert \times \lvert\vec{B}\rvert \times \cos(\theta)$ where θ is the angle between the vectors (b) $[a, b] \cdot [c, d] = ac + bd$ (c) $[a, b, c] \cdot [e, f, g] = ae + bf + cg$
5. LINEAR COMBINATION:	$\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$ if $\vec{w} = k\vec{u} + \ell\vec{v}$, where k and ℓ are scalars.
6. LINEAR COMBINATION THEOREMS:	If $\vec{u} = \overrightarrow{OP}$, $\vec{w} = \overrightarrow{OQ}$ and $\vec{v} = \overrightarrow{OR}$ are three vectors with P, Q and R collinear, and $\overrightarrow{PQ} = k\overrightarrow{QR}$, then $\vec{w} = \frac{k\vec{v} + \vec{u}}{k + \ell}$; Q divides the line segment PR in the ratio $k:\ell$.
7. V_2:	Indicates vector on a plane, given by ordered pairs.
8. V_3:	Indicates vector in space, given by ordered triples.
9. V_2 BASIS:	The natural basis for V_2 is the pair of vectors $\vec{e}_1 = [1, 0]$ and $\vec{e}_2 = [0, 1]$.

TERM	DEFINITION
10. V_3 BASIS:	The natural basis for V_3 is the set $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$, where $\vec{e}_1 = [1, 0, 0]$, $\vec{e}_2 = [0, 1, 0]$, $\vec{e}_3 = [0, 0, 1]$.
11. PROJECTION:	The projection of $\vec{u}$ on $\vec{v}$ is $\frac{\lvert\vec{u} \cdot \vec{v}\rvert}{\lvert v\rvert}$.
12. WORK:	Work = (magnitude of force in the direction of motion) $\times$ (distance).
13. POLAR FORM:	Specifies magnitude and direction, $[\lvert\vec{A}\rvert, \theta^\circ]$.
14. INNER PRODUCT:	Same as dot product.

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